Int J Theor Phys (2009) 48: 1110–1121
DOI 10.1007/s10773-008-9884-x

Effect of the A-Term on the Nucleon Properties in the Extended Linear Sigma Model

M. Abu-Shady

Received: 20 August 2008 / Accepted: 27 October 2008 / Published online: 13 November 2008

Abstract In this paper, the effect of the gradient-expanded meson term on the nucleon properties is investigated. The A-term is included in the extended linear sigma model in the framework of some aspects of QCD. The field equations have been solved in the mean-field approximation for the hedgehog baryon state. We find that the inclusion of the new term improves the description of the nucleon properties in comparison with other models.

Keywords Mean-field approximation · Nucleon properties · Linear sigma model

1 Introduction

It is now becoming clear that the chiral symmetry of the QCD Lagrangian and its spontaneous breaking [1] play a very important role in determining the structure of low-mass hadrons, which consist of u, d and s quarks, and simultaneously play a crucial role in hadron correlators in mediating the spontaneous chiral symmetry breaking [2, 3].

One of the effective models in describing hadron properties is the linear sigma model which has been suggested earlier by Gell-Mann and Levy [4] to describe nucleons interacting via sigma (σ) and pion ($\boldsymbol{\pi}$) exchanges. This model is a principal example of spontaneous symmetry breaking. Some of the consequences of this model, however, are known to be in conflict with observation. Notably, the isoscalar pion-nucleon (πN) scattering length predicted by the model is larger than experimental value by an order of magnitude; see e.g. Refs. [5–8]. Many solutions for this model have already been suggested. Birse and Banerjee [5] constructed equations of motion treating both the σ- and $\boldsymbol{\pi}$-fields as time-independent classical fields and quarks in the hedgehog spinor state. Birse [6] generalized this mean-field model to include angular and momentum and isospin projection. Goeke et al. [7] investigated hadron properties in a chiral model for the nucleon based on the linear sigma model with scalar-isoscalar and scalar-isovector mesons coupled to quarks using the coherent pair

M. Abu-Shady (✉)
Department of Mathematics, Faculty of Science, Menoufia University, Shebin El-Koum, Egypt
e-mail: abu_shady_1999@yahoo.com

approximation. That work has been reexamined by Aly et al. [8]. However, a large error has been obtained in calculating $\sigma(\pi N)$ in comparison with the experimental data.

During the last few years, there has been intense interest in various chiral quark models to the description of nucleon properties, such as the perturbative chiral quark model [9–12], extended Skyrme model [13, 14] and extended linear sigma model [15–17]. The perturbative chiral quark model is an effective model of baryons based on chiral symmetry. The baryon is described as a state of three localized relativistic quarks supplemented by a pseudoscalar meson cloud as dictated by chiral symmetry requirements. In this model the effect of the meson cloud is evaluated perturbatively in a systematic fashion. The model has been successfully applied to the nucleon electro-magnetic form factors, the meson-nucleon sigma term, the nucleon-photo transition and the nucleon axial coupling constant amongst other applications (for details, see Refs. [9–12]). The original Skyrme model Lagrangian consists of the non-linear sigma model term and the fourth-order derivative term, which guarantees the stabilization of the soliton so that the degree of freedom of the sigma field may be replaced by a variable chiral radius, which becomes the new dynamic degree of freedom that plays an important role in the modified Skyrmion Lagrangian density, in which the dependence of the observables on the values of the scalar meson mass and on the fourth-order coupling term e were also considered [13, 14]. On the other hand, Broniowski and Golli [15] analyzed a particular extension of the linear sigma model coupled to valence quarks, which contained an additional term with gradients of the chiral fields and investigated the dynamic consequence of this term and its relevance to the phenomenology of the soliton models of the nucleon. In the same direction, Rashdan et al. [16, 17] considered the higher-order mesonic interactions in the chiral quark sigma model to get a better description of nucleon properties.

The aim of this paper is to estimate the effect of the A-term which was introduced by Broniowski and Golli [15] on the description of nucleon properties in the extended linear sigma model as proposed by Rashdan et al. [16]. Especially, the properties are not calculated in Ref. [15].

The paper is organized as follows. In Sect. 2, the extended sigma model is reviewed briefly. The numerical calculations and the results are presented in Sect. 3.

2 Extended Quark Sigma Model with the A-Term

2.1 Extended Quark Sigma Model

In this subsection, we give a brief summary of the extended chiral quark sigma model which is described in detail in Ref. [16].

The Lagrangian density of the extended sigma model which describes the interactions between quarks via the σ- and $\boldsymbol{\pi}$-mesons is written as in Ref. [16]

$$L(r) = \overline{\Psi} i\gamma_\mu \partial^\mu \Psi + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi}.\partial^\mu \boldsymbol{\pi}\right) + g\overline{\Psi}\left(\sigma + i\gamma_5 \boldsymbol{\tau}.\boldsymbol{\pi}\right)\Psi - U(\sigma, \boldsymbol{\pi}), \tag{1}$$

with

$$U(\sigma, \boldsymbol{\pi}) = \frac{\lambda^2}{4}\left((\sigma^2 + \boldsymbol{\pi}^2)^2 - \nu^2\right)^2 + m_\pi^2 f_\pi \sigma, \tag{2}$$

which is the meson-meson interaction potential where Ψ, σ and $\boldsymbol{\pi}$ are the quark, sigma and pion fields, respectively. In the mean-field approximation, the meson fields are treated as time-independent classical fields. This means that we are replacing powers and products of the meson fields by corresponding powers and products of their expectation values. The meson-meson interactions in (2) lead to hidden chiral $SU(2) \times SU(2)$ symmetry with $\sigma(r)$ taking on a vacuum expectation value

$$\langle \sigma \rangle = -f_\pi, \tag{3}$$

where $f_\pi = 93$ MeV is the pion decay constant. The final term in (2) is included to break the chiral symmetry. It leads to partial conservation of axial-vector isospin current (PCAC). The parameters λ^2, ν^2 are expressed in terms of f_π and the masses σ- and $\boldsymbol{\pi}$-mesons:

$$\lambda^2 = \frac{m_\sigma^2 - 3m_\pi^2}{8 f_\pi^6}, \tag{4}$$

$$\nu^2 = f_\pi^4 - \frac{m_\pi^2}{2\lambda^2 f_\pi^2} \tag{5}$$

(for details, see Ref. [16]).

2.2 The Mesonic A-Term

The general form for the mesonic Lagrangian is given by [15]

$$L_{mes} = -U_1 + \frac{1}{2} Z \left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} . \partial^\mu \boldsymbol{\pi}\right) + \frac{1}{2} A \left(\sigma \partial^\mu \sigma + \boldsymbol{\pi} . \partial^\mu \boldsymbol{\pi}\right)^2 \tag{6}$$

where Z and A are arbitrary functions of the invariant $(\sigma^2 + \boldsymbol{\pi}^2)$. The last term in (6) is called the A-term, which is put in the framework of some aspects of QCD.

We can write the new Lagrangian as in Ref. [15]

$$\begin{aligned} L(r) = {} & \overline{\Psi} i \gamma_\mu \partial^\mu \Psi + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} . \partial^\mu \boldsymbol{\pi}\right) + \frac{1}{2} A_0 \left(\sigma \partial^\mu \sigma + \boldsymbol{\pi} . \partial^\mu \boldsymbol{\pi}\right)^2 \\ & + g \overline{\Psi} \left(\sigma + i \gamma_5 \boldsymbol{\tau} . \boldsymbol{\pi}\right) \Psi - U_1(\sigma, \boldsymbol{\pi}), \end{aligned} \tag{7}$$

with

$$U_1(\sigma, \boldsymbol{\pi}) = \frac{\lambda_1^2}{4}\left((\sigma^2 + \boldsymbol{\pi}^2)^2 - \nu_1^2\right)^2 + m_\pi^2 f_\pi \sigma, \tag{8}$$

where

$$\lambda_1^2 = \frac{\bar{m}_\sigma^2 - 3m_\pi^2}{8 f_\pi^6}, \tag{9}$$

$$\nu_1^2 = f_\pi^4 - \frac{m_\pi^2}{2\lambda_1^2 f_\pi^2}, \tag{10}$$

$$\bar{m}_\sigma^2 = \left(1 + f_\pi^2 A_0\right) m_\sigma^2. \tag{11}$$

(For details, see Ref. [15].)

Now we expand the extremum, with the shifted field defined as

$$\sigma = \sigma' - f_\pi , \tag{12}$$

substituting by (12) into (7), we get

$$L(r) = \overline{\Psi} i\gamma_\mu \partial^\mu \Psi + \frac{1}{2}\left(\partial_\mu \sigma' \partial^\mu \sigma' + \partial_\mu \boldsymbol{\pi}.\partial^\mu \boldsymbol{\pi}\right) + \frac{1}{2} A_0 \left(\sigma' \partial^\mu \sigma' + \boldsymbol{\pi}.\partial^\mu \boldsymbol{\pi}\right)^2 - g\overline{\Psi} f_\pi \Psi$$
$$+ g\overline{\Psi}\sigma'\Psi + ig\overline{\Psi}\boldsymbol{\gamma}_5.\boldsymbol{\pi}\Psi - U_1\left(\sigma', \boldsymbol{\pi}\right) \tag{13}$$

with

$$U_1\left(\sigma', \boldsymbol{\pi}\right) = \frac{\lambda_1^2}{4}\left(((\sigma' - f_\pi)^2 + \boldsymbol{\pi}^2)^2 - \nu_1^2\right)^2 + m_\pi^2 f_\pi (\sigma' - f_\pi).$$

The time-independent fields $\sigma'(r)$ and $\boldsymbol{\pi}(r)$ satisfy the Euler-Lagrange equation, and the quark wave function satisfies the Dirac eigenvalue equation. Substituting by (13) in the Euler-Lagrange equation as in Ref. [15], we obtain

$$\Box\sigma' = \frac{-1}{(1 + ((\sigma' - f_\pi)^2 + \boldsymbol{\pi}^2)A_0)}\Bigg\{(1 + \boldsymbol{\pi}^2 A_0)\Bigg(A_0(\sigma' - f_\pi)((\partial^\mu \sigma')^2$$
$$+ (\partial^\mu \boldsymbol{\pi})^2) + \frac{\partial U_1}{\partial \sigma'} - g\overline{\Psi}\Psi\Bigg)$$
$$- A_0(\sigma' - f_\pi)\boldsymbol{\pi}\Bigg(A_0\boldsymbol{\pi}((\partial^\mu \sigma')^2 + (\partial^\mu \boldsymbol{\pi})^2) + \frac{\partial U_1}{\partial \boldsymbol{\pi}} - ig\overline{\Psi}\gamma_5.\boldsymbol{\tau}\Psi\Bigg)\Bigg\}, \tag{14}$$

$$\Box\boldsymbol{\pi} = \frac{-1}{(1 + ((\sigma' - f_\pi)^2 + \boldsymbol{\pi}^2)A_0)}\Bigg\{(-A_0(\sigma' - f_\pi)\boldsymbol{\pi})\Bigg(A_0(\sigma' - f_\pi)((\partial^\mu \sigma')^2$$
$$+ (\partial^\mu \boldsymbol{\pi})^2) + \frac{\partial U_1}{\partial \sigma'} - g\overline{\Psi}\Psi\Bigg)$$
$$+ (1 + (\sigma' - f_\pi)^2 A_0)\Bigg(A_0\boldsymbol{\pi}((\partial^\mu \sigma')^2 + (\partial^\mu \boldsymbol{\pi})^2) + \frac{\partial U_1}{\partial \boldsymbol{\pi}} - ig\overline{\Psi}\gamma_5.\boldsymbol{\tau}\Psi\Bigg)\Bigg\}, \tag{15}$$

where $\boldsymbol{\tau}$ refers to Pauli isospin matrices and $\gamma_5 = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$. If $A_0 = 0$, the usual extended linear sigma model [16] and the equations of motion are recovered. Including the color degree of freedom, one has $g\overline{\Psi}\Psi \to N_c g\overline{\Psi}\Psi$ where $N_c = 3$ colors. Thus

$$\Psi(r) = \frac{1}{\sqrt{4\pi}}\begin{bmatrix} u(r) \\ iw(r) \end{bmatrix} \quad \text{and} \quad \bar{\Psi}(r) = \frac{1}{\sqrt{4\pi}}\begin{bmatrix} u(r) & iw(r) \end{bmatrix}, \tag{16}$$

and the sigma, pion and vector densities are given by

$$\rho_s = N_c\overline{\Psi}\Psi = \frac{3}{4\pi}\left(u^2 - w^2\right), \tag{17}$$

$$\rho_p = iN_c\overline{\Psi}\gamma_5\vec{\tau}\Psi = \frac{3}{4\pi}g(-2uw), \tag{18}$$

$$\rho_v = \frac{3}{4\pi}\left(u^2 + w^2\right). \tag{19}$$

The boundary conditions for the asymptotics for $\sigma(r)$ and $\pi(r)$ at $r \to \infty$ are:

$$\sigma\,(r) \sim -f_\pi, \qquad \pi\,(r) \sim 0. \tag{20}$$

We used the hedgehog ansatz [5], where

$$\boldsymbol{\pi}\,(r) = \hat{\mathbf{r}}\pi\,(r)\,. \tag{21}$$

The chiral Dirac equation for the quarks is [16]

$$\frac{du}{dr} = -P\,(r)\,u + \big(W + m_q - S(r)\big)\,w, \tag{22}$$

where the scalar potential $S(r) = g\langle\sigma'\rangle$, the pseudoscalar potential $P(r) = \langle\boldsymbol{\pi}.\hat{r}\rangle$, and W is the eigenvalue of the quarks spinor Ψ.

$$\frac{dw}{dr} = -\big(W - m_q + S(r)\big)\,u - \left(\frac{2}{r} - P\,(r)\right)w. \tag{23}$$

3 Numerical Calculations

3.1 The Scalar Field σ'

To solve (14), we integrate a suitable Green's function over the source fields as in Ref. [18]. Thus

$$\begin{aligned}\sigma'(\mathbf{r}) = \int d^3\mathbf{r}' D_\sigma(\mathbf{r} - \grave{\mathbf{r}})\Bigg[\frac{-1}{(1 + ((\sigma' - f_\pi)^2 + \boldsymbol{\pi}^2)A_0)}\Bigg\{(1 + \boldsymbol{\pi}^2 A_0)\Bigg(A_0(\sigma' - f_\pi)((\partial^\mu\sigma')^2 \\ + (\partial^\mu\boldsymbol{\pi})^2) + \frac{\partial U_1}{\partial\sigma'} - g\rho_s(\grave{\mathbf{r}})\Bigg) - A_0(\sigma' - f_\pi)\boldsymbol{\pi}\Bigg(A_0\boldsymbol{\pi}((\partial^\mu\sigma')^2 + (\partial^\mu\boldsymbol{\pi})^2) \\ + \frac{\partial U_1}{\partial\boldsymbol{\pi}} - g\rho_p(\grave{\mathbf{r}})\Bigg)\Bigg\}\Bigg]\end{aligned} \tag{24}$$

where

$$D_\sigma(\mathbf{r} - \grave{\mathbf{r}}) = \frac{1}{4\pi|\mathbf{r} - \grave{\mathbf{r}}|}\exp(-m_\sigma|\mathbf{r} - \grave{\mathbf{r}}|),$$

the scalar field is spherical in this model as we only need the $l = 0$ term

$$D_\sigma\left(\mathbf{r} - \grave{\mathbf{r}}\right) = \frac{1}{4\pi}\sinh\left(m_\sigma r_<\right)\frac{\exp\left(-m_\sigma r_>\right)}{r_>}, \tag{25}$$

therefore we arrive at the integral equation for $\sigma'(\mathbf{r})$:

$$\begin{aligned}\sigma'(\mathbf{r}) = m_\sigma\int_0^\infty r'^2 dr'\left(\frac{\sinh\left(m_\sigma r_>\right)}{m_\sigma r_>}\frac{\exp\left(-m_\sigma r_>\right)}{m_\sigma r_>}\right)\Bigg[\frac{-1}{(1 + ((\sigma' - f_\pi)^2 + \boldsymbol{\pi}^2)A_0)} \\ \times\Bigg\{(1 + \boldsymbol{\pi}^2 A_0)\Bigg(A_0(\sigma' - f_\pi)((\partial^\mu\sigma')^2 + (\partial^\mu\boldsymbol{\pi})^2) + \frac{\partial U_1}{\partial\sigma'} - g\rho_s(\grave{\mathbf{r}})\Bigg) \\ - A_0(\sigma' - f_\pi)\boldsymbol{\pi}\Bigg(A_0\boldsymbol{\pi}((\partial^\mu\sigma')^2 + (\partial^\mu\boldsymbol{\pi})^2) + \frac{\partial U_1}{\partial\boldsymbol{\pi}} - g\rho_p(\grave{\mathbf{r}})\Bigg)\Bigg\}\Bigg].\end{aligned} \tag{26}$$

We will solve these implicit integral equation by iterating to self-consistency.

3.2 The Pion Field $\boldsymbol{\pi}$

To solve (15), we integrate a suitable Green's function over the source fields. We use the $l=1$ component of the pion Green's function. Thus

$$\begin{aligned}\boldsymbol{\pi}\,(r) &= m_\pi \int_0^\infty r'^2 dr' \frac{[-\sinh(m_\pi r_<) + m_\pi r_< \cosh(m_\pi r_<)]}{(m_\pi r_>)^2}\\ &\quad\times \frac{-1}{(1+((\sigma'-f_\pi)^2+\boldsymbol{\pi}^2)A_0)}\Big\{-A_0(\sigma'-f_\pi)\Big(\boldsymbol{\pi} A_0(\sigma'-f_\pi)((\partial^\mu\sigma')^2+(\partial^\mu\boldsymbol{\pi})^2)\\ &\quad+\frac{\partial U_1}{\partial\sigma'} - g\rho_s(\grave{\mathbf{r}})\Big) + (1+(\sigma'-f_\pi)^2A_0)\Big(A_0\boldsymbol{\pi}((\partial^\mu\sigma')^2+(\partial^\mu\boldsymbol{\pi})^2)\\ &\quad+\frac{\partial U_1}{\partial\boldsymbol{\pi}} - g\rho_p(\grave{\mathbf{r}})\Big)\Big\}. \end{aligned} \tag{27}$$

We have solved Dirac equations (22), (23) using the fourth-order Runge Kutta method. Due to the implicit nonlinearly of (14), (15), it is necessary to iterate the solution until self-consistency is achieved. To start this iteration process, we use the chiral circle form for the meson fields:

$$S(r) = m_q(1-\cos\theta) \quad \text{and} \quad P(r) = -m_q\sin\theta, \tag{28}$$

where $\theta = \tanh r$ and m_q is the quark mass.

3.3 Properties of the Nucleon

3.3.1 The Nucleon Mass

The energy density ε is given by

$$\varepsilon = T^{00} = \frac{\partial L}{\partial\,(\partial_0\Phi_i)}\partial_0\Phi_i - L \tag{29}$$

where T^{00} can be written as

$$\begin{aligned}T^{00} &= i\overline{\Psi}\partial^0\gamma^0\Psi + \partial_0\sigma'\partial^0\sigma' + \partial_0\boldsymbol{\pi}\cdot\partial^0\boldsymbol{\pi} - \Big[\overline{\Psi}i\gamma^\mu\partial_\mu\Psi - gf_\pi\overline{\Psi}\Psi\\ &\quad+ igf_\pi\overline{\Psi}\gamma^5\boldsymbol{\tau}\Psi.\boldsymbol{\pi} + g\overline{\Psi}\Psi\sigma' + \frac{1}{2}\left(\partial_\mu\sigma'\right)^2 + \frac{1}{2}\left(\partial_\mu\boldsymbol{\pi}\right)^2 - U_1(\sigma',\boldsymbol{\pi})\Big],\end{aligned} \tag{30}$$

so

$$\begin{aligned}\varepsilon &= -\overline{\Psi}(i\boldsymbol{\nabla}.\boldsymbol{\gamma}+m_q)\Psi + \frac{1}{2}\left(\boldsymbol{\nabla}\sigma'\right)^2 + \frac{1}{2}(\nabla.\boldsymbol{\pi})^2\\ &\quad- g\overline{\Psi}(i\gamma_5\boldsymbol{\tau}.\boldsymbol{\pi}+\sigma')\Psi + U_1(\sigma',\boldsymbol{\pi}) - U_1(\sigma'=0,\boldsymbol{\pi}=0),\end{aligned} \tag{31}$$

which represents the quark, σ' and $\boldsymbol{\pi}$ kinetic energy, quark-meson and meson-meson interaction terms, respectively.

The kinetic energy terms for a quark can be reexpressed via the Dirac eigenvalue equation of motion

$$(K.E)_{quark} = -(m_q - S(r))\rho_s(r) + W\rho_v(r) + P(r)\rho_p(r). \tag{32}$$

We have the kinetic equation for the meson field

$$(K.E)_{sigma} = \frac{1}{2}\sigma' \Box \sigma'. \tag{33}$$

Similarly for the pion field, we have

$$(K.E)_{pion} = \frac{1}{2}\boldsymbol{\pi} \Box \boldsymbol{\pi}. \tag{34}$$

The meson static energy is given by

$$E_{Static} = U_1(\sigma', \boldsymbol{\pi}) - U_1(\sigma' = 0, \boldsymbol{\pi} = 0). \tag{35}$$

The sigma-quark interaction energy and the sigma-pion interaction energy are given respectively by

$$\left(m_q - g\sigma'\right)\rho_s \quad \text{and} \quad -g\pi\rho_p. \tag{36}$$

Finally

$$\begin{aligned} E_{total} &= \int d^3 r \varepsilon(r) \\ &= N_c W + 4\pi \int_0^\infty r^2 dr \varepsilon(r). \end{aligned} \tag{37}$$

(For details, see Ref. [16].)

3.3.2 *Magnetic Moment, Axial Vector Coupling Constant* $g_A(0)$ *and Pion-Nucleon Coupling Constant* $\frac{g_{\pi NN}(0)}{2M_N}$

The magnetic moments of the proton and neutron are given by

$$\mu_{p,n} = \langle P \uparrow | \int d^3\mathbf{r} \frac{1}{2}\mathbf{r} \times \mathbf{j}_{\varepsilon M}(\mathbf{r}) | P \uparrow \rangle, \tag{38}$$

where the electromagnetic current is

$$j_{\epsilon M}(\mathbf{r}) = \bar{\Psi}(\mathbf{r}) \boldsymbol{\gamma} \left(\frac{1}{6} + \frac{\tau_3}{2}\right) \Psi(\mathbf{r}) - \varepsilon_{\alpha\beta_3} \pi_\alpha(\mathbf{r}) \nabla \pi_\beta(\mathbf{r}), \tag{39}$$

such that

$$(\mathbf{j}_{\epsilon M}(\mathbf{r}))_{nucleon} = \bar{\Psi}(\mathbf{r}) \boldsymbol{\gamma} \left(\frac{1}{6} + \frac{\tau_3}{2}\right) \Psi(\mathbf{r}), \tag{40}$$

$$(\mathbf{j}_{\epsilon M}(\mathbf{r}))_{meson} = -\epsilon_{\alpha\beta 3} \pi_\alpha(\mathbf{r}) \nabla \pi_\beta(\mathbf{r}). \tag{41}$$

The nucleon axial-vector coupling constant is found from

$$\frac{1}{2} g_A(0) = \langle P \uparrow | \int d^3 r A_3^z(\mathbf{r}) | P \uparrow \rangle, \tag{42}$$

Table 1 Values of nucleon mass, nucleon magnetic moments, $\sigma(\pi N)$ term, $g_A(0)$ and $\frac{g_{\pi NN}(0)}{2M_N}$ at $f_\pi^2 A_0 = -0.03$ and $m_\sigma = 441$ MeV. All quantities in MeV

m_q	440	460	480	Expt. [5, 13]
M_n	964	926	880	938
$\mu_p(N)$	2.719	2.767	2.81	2.79
$\mu_n(N)$	−2.00	−2.06	−2.21	−1.91
$\sigma(\pi N)$	110	111	112	50 ± 20
$g_A(0)$	1.71	1.73	1.74	1.25
$\frac{g_{\pi NN}(0)}{2M_N}$	1.423	1.464	1.502	1.0

where the z-component of the axial vector current is given by

$$A_3^z(\mathbf{r}) = \bar{\Psi}(\mathbf{r}) \frac{1}{2}\gamma_5\gamma^3\tau_3\Psi(\mathbf{r}) - \sigma(\mathbf{r}) \frac{\partial}{\partial z}\pi_3(\mathbf{r}) + \pi_3(\mathbf{r}) \frac{\partial}{\partial z}\sigma(\mathbf{r}). \tag{43}$$

The pion-nucleus σ commutator is defined

$$\sigma(\pi N) = \langle P \uparrow | \int d^3 r \sigma'(\mathbf{r}) | P \uparrow \rangle. \tag{44}$$

In the calculation of $\sigma(\pi N)$, we replace $\sigma'(\mathbf{r})$ by $\frac{j_\sigma(\mathbf{r})}{m_\sigma^2}$ where $j_\sigma(\mathbf{r})$ is the source current defined by

$$(\Box + m_\sigma^2)\sigma' = j_\sigma(\mathbf{r}).$$

To calculate the pion-nucleon coupling constant, we consider the limit $\mathbf{q} \longrightarrow 0$ of

$$\frac{g_{\pi NN}(0)}{2M} = \langle P \uparrow | \int d^3\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \times \mathbf{j}_{\pi 3}(\mathbf{r}) | P \uparrow \rangle, \tag{45}$$

where the pion source current is defined by

$$(\Box + m_\pi^2) = \mathbf{j}_{\pi 3}(\mathbf{r}). \tag{46}$$

(For details, see Refs. [5, 6].)

3.4 Discussion of Results

The field equations (14)→(23) have been solved by the iteration method as in Refs. [16, 17] for different values of quark and sigma masses which are constituent with the quark models in Refs. [19–21].

Table 1 represents the results for sigma mass $m_\sigma = 441$ MeV as predicted by chiral perturbation theory [18]. Note that the nucleon mass is reduced by the increase of quark mass or for high values of g ($m_q = g f_\pi$). This is due to the increase in the attractive interaction between quarks and mesons which leads to a decrease in the nucleon mass. On the inverse, the sigma commutator $\sigma(\pi N)$, coupling constant $g_A(0)$ and pion-nucleon coupling constant $\frac{g_{\pi NN}(0)}{2M_N}$ are increased by increasing the quark mass.

From Table 2, the results are calculated for $m_\sigma = 650$ MeV, which are consistent with Ref. [19]. Note that the situation is similar to the situation described in Table 1 but is a little different in the results due to the increase in the sigma mass.

Table 2 Values of nucleon mass, magnetic moments of the nucleon, $\sigma(\pi N)$ term, $g_A(0)$ and $\frac{g_{\pi NN}(0)}{2M_N}$ at $f_\pi^2 A_0 = -0.03$ and $m_\sigma = 650$ MeV. All quantities in MeV

m_q	440	460	480	Expt. [5, 13]
M_n	994	949	897	938
$\mu_p(N)$	2.662	2.726	2.78	2.79
$\mu_n(N)$	−1.99	−2.069	−2.13	−1.91
$\sigma(\pi N)$	90	92	93	50 ± 20
$g_A(0)$	1.732	1.754	1.77	1.25
$\frac{g_{\pi NN}(0)}{2M_N}$	1.40	1.447	1.48	1.0

Table 3 Values of nucleon mass, magnetic moments of the nucleon, $\sigma(\pi N)$ term, $g_A(0)$ and $\frac{g_{\pi NN}(0)}{2M_N}$ at $f_\pi^2 A_0 = -0.03$ and $m_\sigma = 1100$ MeV. All quantities in MeV

m_q	440	460	480	Expt. [5, 13]
M_n	1046	994	938	938
$\mu_p(N)$	2.569	2.616	2.647	2.79
$\mu_n(N)$	−1.980	−2.037	−2.07	−1.91
$\sigma(\pi N)$	64	67	69	50 ± 20
$g_A(0)$	1.75	1.77	1.78	1.25
$\frac{g_{\pi NN}(0)}{2M_N}$	1.32	1.35	1.38	1.0

From Table 3, the results are calculated for sigma mass $m_\sigma = 1100$ MeV. Better results are obtained for ($m_\sigma = 1100$, $m_q = 480$) MeV. This set of masses and coupling constant predicted the correct nucleon mass (after subtracting the CM correction), where we get a value 938 MeV, as seen from Table 3 that due to the inclusion of the A-term since it lowers the energy of solitons for high values of g [15]. This table also shows that the experimental magnetic moments of the proton and neutron are well reproduced. The Goldberger-Trieman pion-nucleon coupling constant $\frac{g_{\pi NN}(0)}{2M}$, which is of the order 1.3, is also reproduced well in comparison with Ref. [5]. The sigma commutator $\sigma(\pi N)$ is improved in range of 24% in comparison with Refs. [5, 16]. We know that the largest value of the sigma commutator is one of the problems in the linear sigma model as in Refs. [5–8]. The A-term has a strong effect on this quantity.

The coupling constant $g_A(0)$ is improved by about 6% but still largest with data which due to the expression of the axial vector current (see (43)) and does not depend explicitly on of the masses, but only on the dynamic of fields [15].

From Fig. 1, we see the sigma field passes through zero at $r = 0.5$, which we will refer to as the soliton radius ($r = 0.5$), whereas at $r \to \infty$ the pion field $\to 0$ and sigma field $\to -f_\pi$. The pion field takes the shape of the P-wave, which gives the attraction of the pion-quark interaction, and goes to zero in a linear manner for large distances. We also see that the meson fields do not stray far from the circular minimum of the potential $\sigma^2 + \boldsymbol{\pi}^2 = f_\pi^2$. The pion field reaches its maximum value close to the soliton radius. Figure 2 shows the components of quarks $u(r)$ and $w(r)$ corresponding the fields.

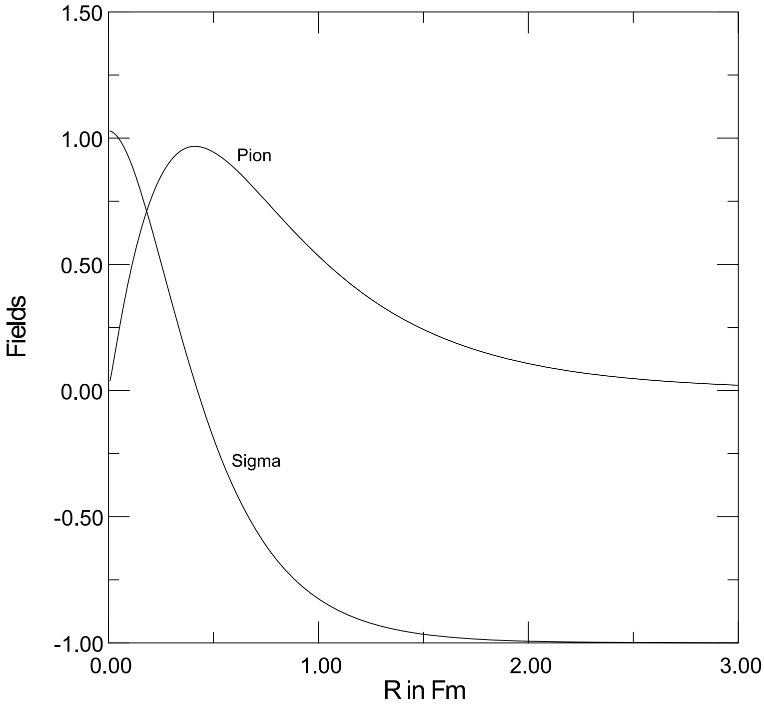

Fig. 1 Sigma and pion fields in units of f_π a function of distance R for $m_\sigma = 1100$ MeV, $m_q = 480$ MeV, $f_\pi^2 A_0 = -0.03$

4 Comparison with Other Models

It is interesting to compare the nucleon properties in the present approach with other models. Here we consider four models: the perturbative chiral quark model [9–12], extended Skyrme model [13], extended sigma model [16] and linear sigma model [5]. The perturbative chiral quark model is an effective model of baryons based on chiral symmetry. The baryon is described as a state of three localized relativistic quarks supplemented by a pseudoscalar meson cloud as dictated by chiral symmetry requirements. The model has been successfully applied to the nucleon properties (see Table 3). We obtained better results for nucleon mass and nucleon magnetic moments and reasonable results for the sigma term $\sigma(\pi N)$ and the coupling constant $g_A(0)$ in comparison with this model which backs to the perturbative chiral quark model based on non-linear σ-model Lagrangian. In comparison with extended Skyrme model [13], we see the predicted nucleon mass, nucleon magnetic moments are improved in comparison with this model. Also, the sigma term $\sigma(\pi N)$ and neutron magnetic moment are improved in comparison with the extended linear sigma model [16].

In comparison with the original sigma model [5], we see the A-term has a strong effect on all observables of the nucleon and all observables are improved due the inclusion of the A-term.

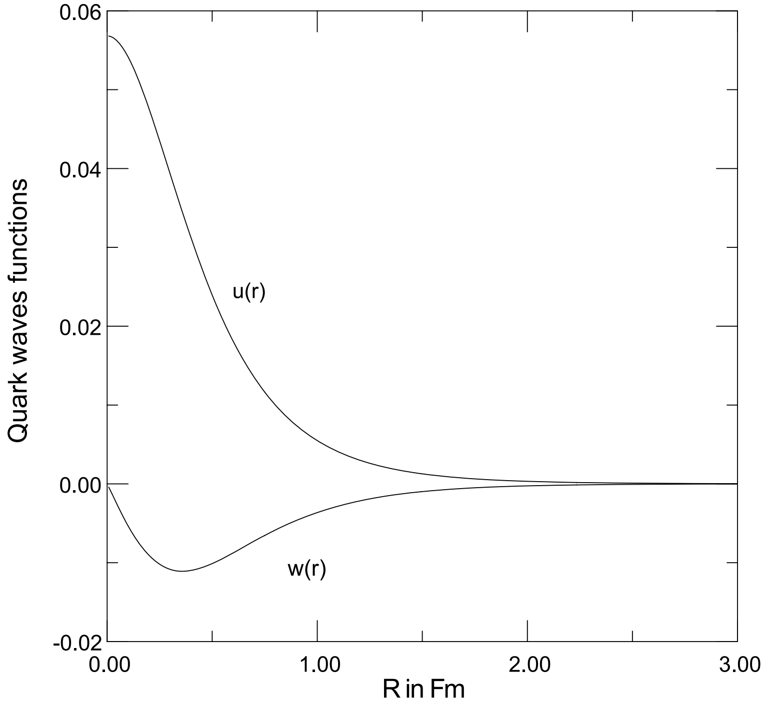

Fig. 2 The components of the quark as function of distance R for $m_\sigma = 1100$ MeV, $m_q = 480$ MeV, $f_\pi^2 A_0 = -0.03$

Table 4 Observables of the nucleon in comparison with linear sigma model [5, 22], extended sigma model [16], perturbative chiral quark model [9–12] and extended Skyrme model [13]

Quantity	Present work	[5, 22]	[16]	[9–12]	[13]	Exp. [13]
M_n	938	993	–	828.5	1436	938
$\mu_p(N)$	2.65	2.87	2.787	2.62 ± 0.02	2.9	2.79
$\mu_n(N)$	−2.07	−2.29	−2.19	-2.0 ± 0.02	−2.1	−1.91
$\sigma(\pi N)$	69	92	88	54.7	54	50 ± 20
$g_A(0)$	1.78	1.86	1.80	1.19	0.78	1.25
$\frac{g_{\pi NN}(0)}{2M_N}$	1.38	1.53	1.1	–	0.61	1.0

5 Conclusion

From the results, the A-term has a strong effect on all observables of the nucleon. In particular, nucleon mass, nucleon magnetic moments and the sigma commutator $\sigma(\pi N)$ are improved in comparison with other models and are in good agreement with measured data. $g_A(0)$ is improved, but has little sensitivity to the A-term.

Acknowledgement This work is financed by the government of Egypt. I would like to thank Prof. A. Faessler for the hospitality during at my stay at Tubingen University.

References

1. Koch, V.: Int. J. Mod. Phys. E **6**, 203 (1997) and references therein
2. Schafer, T., Evshuryak: Rev. Mod. Phys. **70**, 323 (1998)
3. Diakanov, D.: hep-ph/9602375, talk given at International School of Physics, 'Enrico Fermi', course 80 (Varenna, Italy, 1995)
4. Gell'mann, M., Levy, M.: Nuovo Cim. **16**, 705 (1960)
5. Birse, M., Banerjee, M.: Phys. Rev. D **31**, 118 (1985)
6. Birse, M.: Phys. Rev. D **33**, 1934 (1986)
7. Goeke, K., Harve, M., Grummer, F., Urbano, J.N.: Phys. Rev. D **37**, 754 (1988)
8. Aly, T.S.T., McNeil, J.A., Pruess, S.: Phys. Rev. D **60**, 1114022 (1999)
9. Lyubovitskij, V.E., Gutsche, Th., Faessler, A.: Phys. Rev. C **64**, 065203 (2001)
10. Inoue, T., Lyubovitskij, V.E., Gutsche, T., Faessler, A.: Phys. Rev. C **69**, 035207 (2004)
11. Khosonthongkee, K., Lyubovitskij, V.E., Gutsche, Th., Faessler, A., Pumsa-ard, K., Cheedket, S., Yan, Y.: J. Phys. G **30**, 793 (2004)
12. Inoue, T., Lyubovitskij, V.E., Gutsche, Th., Faessler, A.: Int. J. Mod. Phys. E **15**, 121 (2006)
13. Braghin, F.L., Cavalcante, I.P.: Phys. Rev. C **67**, 065207 (2003)
14. Braghin, F.L.: J. Braz. Phys. **34**(1A) (2004)
15. Broniowski, W., Golli, B.: Nucl. Phys. A **714**, 575–588 (2003)
16. Rashdan, M., Abu-shady, M., Ali, T.S.T.: Int. J. Mod. Phys. E **15**, 143–153 (2006)
17. Rashdan, M., Abu-Shady, M., Ali, T.S.T.: Int. J. Mod. Phys. A **22**(14 & 15), 2673 (2007)
18. Koonin, S.E., Meredith, D.C.: Computational Physics (Fortran version). Addison-Wesley, Reading (1990)
19. Leutwyler, H.: Int. Mod. Phys. A **22**, 257–265 (2007)
20. van Beveren, E., Kleefeld, F., Rupp, G., Scadron, M.D.: Mod. Phys. Lett. A **17**, 1673 (2002)
21. Rashdan, M.: Chaos, Solitons Fractals **18**, 107 (2003)
22. Cohen, T.D., Broniowski, W.: Phys. Rev. D **34**, 3472 (1986)